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A New Aperture Admittance Model for Open-Ended Waveguides

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Abstract

A new model for the aperture admittance of open-ended waveguide structures radiating into a homogeneous, lossy dielectric is presented. The model is based on the physical and mathematical properties of the driving point admittance of passive, stable one-port networks. The model parameters, which depend upon the geometry of the waveguide and aperture, are determined from a relatively small number of computed admittances. This computed data is obtained by a full-wave moment method solution and, hence, includes the effects of radiation and energy storage in the near-field and the evanescent waveguide modes. The accuracy of the numerical method is demonstrated by comparison with measured values. As an example, the model parameters are determined for the coaxial-line geometry. The accuracy of the model, for both the direct and inverse problem, is verified. The new model has important applications in the field of dielectric spectroscopy.

I. Introduction

The increasing use of microwaves and millimeter waves in such diverse fields as communications, radar, medicine, biology, agriculture and industrial processes demands accurate data on the dielectric properties of materials. Classical techniques of dielectric spectroscopy [1,2,3] generally involve measuring the complex impedance, or the complex resonant frequency, of a two terminal structure containing the material under test. These techniques require extensive sample preparation which, in many applications, is not practical.

A convenient system for the non-destructive measurement of material permittivity consists of an open-ended waveguide[1] sensor and a network analyzer. The sensor is placed in contact with the material under test and the reflection coefficient is measured. Knowledge of the relationship between the measured reflection coefficient (Γ) and the permittivity (ε) then allows one to determine the latter. Although accurate numerical methods exist for the calculation of Γ for a given ε, in practice one is interested in the inverse problem. Iterative inversion, based on these numerical methods, are time consuming and yield no information regarding the measurement uncertainty. Ideally, a closed form expression for the permittivity as a function of the reflection coefficient is required. Some attempt has been made to this end in the case of open-ended coaxial lines [4,5,6,7,8,9,10]. The results, however, are based on static or quasi-static approximations, and are valid only for restricted frequencies.

1 The term waveguide is used in it's most general sense to include transmission lines as well as hollow pipes.

This paper presents a general technique for obtaining accurate, broadband, aperture admittance models for open-ended waveguides in contact with homogeneous dielectrics. The model is explicit and easily inverted.

II. Theory

Consider a uniform waveguide terminated in the plane z=0 by a perfectly conducting screen containing the aperture (Fig. 1). The guide is excited by the dominant mode and the frequency of operation is low enough to ensure single-mode propagation. The half-space, z>0, is filled with a homogeneous, linear, isotropic, non-magnetic dielectric with relative permittivity $\varepsilon_r = \varepsilon' - j\varepsilon''$. In this case, one can define the normalized aperture admittance in terms of the dominant mode reflection coefficient at the aperture by:

$$Y = \frac{1-\Gamma}{1+\Gamma} = \frac{1}{Y_0}\frac{H_{a0}}{E_{a0}} \tag{1}$$

Where H_{a0} is the amplitude of the total (incident+reflected) tangential magnetic field in the aperture, E_{a0} is the corresponding electric field, Γ is the dominant mode reflection coefficient, and Y_0 is the dominant mode wave admittance in the guide.

For any given waveguide and aperture geometry, this admittance is a complex function of two complex variables,

$$Y(s,\varepsilon_r) = G(s,\varepsilon_r) + jB(s,\varepsilon_r) \tag{2}$$

where $s = \sigma + j\omega$ is the complex frequency variable, G is the normalized conductance, and B is the normalized susceptance. For any passive dielectric, this admittance must satisfy the same physical requirements as the driving-point admittance of any passive one-port network [11] The mathematical implications of these physical properties are conveniently summarized by stating that the aperture admittance must be a positive-real (p.r.) function in the complex frequency plane. [11] This guarantees the existence of a rational function expansion of the form:

$$Y(s,\varepsilon_r) = \frac{\sum_{n=0}^{\infty} a_n(\varepsilon_r) s^n}{1 + \sum_{m=1}^{\infty} b_m(\varepsilon_r) s^m} \tag{3}$$

The final step is to model the dependance of the coefficients a_n and b_m on the permittivity. The p.r. character of the aperture admittance and the fact that, for steady state and passive dielectrics, the admittance must be an analytic function of ε may

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be used to show that the coefficient a_0 is a function of the D.C. conductivity of the material, independent of the permittivity. If we consider only materials with no D.C. conductivity, this term vanishes.

The remainder of the coefficients certainly depend upon the permittivity. The p.r character of the admittance requires:

$$\Im\{Y(s)\}|_{\omega=0}=0 \qquad (4)$$

$$\Re\{Y(s)\}|_{\omega=0,\sigma>0}>0 \qquad (5)$$

$$Y(s^*)=Y^*(s) \qquad (6)$$

Where $\Re$ and $\Im$ denote the real and imaginary part of a complex quantity, respectively, and * denotes complex conjugation. In addition, the permittivity of passive dielectrics is also a p.r. function of frequency. Defining,

$$\zeta(s)=\sqrt{\varepsilon_r(s)} \qquad (7)$$

where the principal value of the square root is implied, one may show that:

$$\Im\{\zeta(s)\}|_{\omega=0}=0 \qquad (8)$$

$$\Re\{\zeta(s)\}|_{\omega=0,\sigma>0}>0 \qquad (9)$$

$$\zeta(s^*)=\zeta^*(s) \qquad (10)$$

Substituting (3) in (4)-(6) and imposing conditions (8)-(10), one concludes that the coefficients $a_n(\zeta)$ and $b_n(\zeta)$ must be p.r. for passive dielectrics. This requires the coefficients to be analytic in the right-half of the ζ-plane. Therefore, Taylor's theorem is applicable and one may write,

$$a_n=\sum_{p=0}^{\infty}\alpha_{np}(\zeta-c)^p \qquad n=1,2,\ldots,\infty \qquad (11)$$

$$b_m=\sum_{q=0}^{\infty}\beta_{mq}(\zeta-c)^p \qquad m=1,2,\ldots,\infty \qquad (12)$$

where the coefficients α_{np} and β_{mq} are real, and c is any point in the right-half plane. These series converge uniformly for all points which lie in the largest open-disk with center c which lies in the right-half plane.

When the wavenumber in the external medium vanishes, the admittance must likewise vanish. This requires $\alpha_{n0}=0$ for all n. Substitution of (11) and (12) into (3), truncating the sums, and absorbing the various powers of c into the coefficients yields the final form of the approximating function:

$$Y(s,\varepsilon_r)\approx\frac{\sum_{n=1}^{N}\sum_{p=1}^{P}\alpha_{np}\zeta^p s^n}{1+\sum_{m=1}^{M}\sum_{q=1}^{Q}\beta_{mq}\zeta^q s^m} \qquad (13)$$

Finally, from the theory of electromagnetic scale models [12], the parameters of (13) may be normalized such that the same expression is valid for a given class of structures. For example, the expression for open-ended, 50 ohm, Teflon-filled coaxial lines is:

$$Y(sa,\varepsilon_r)\approx\frac{\sum_{n=1}^{N}\sum_{p=1}^{P}\hat{\alpha}_{np}\zeta^p(sa)^n}{1+\sum_{m=1}^{M}\sum_{q=1}^{Q}\beta_{mq}\zeta^q(sa)^m} \qquad (14)$$

where a represents the radius of the inner conductor.

With a suitable model available, the approximation problem becomes one of parametric modelling. This is accomplished by a non-linear least squares fit (Levenberg-Marquardt) to the admittance computed via the method of moments.

III. Results

To illustrate the technique, an admittance model for 50 ohm, open-ended, Teflon-filled coaxial lines is developed. A moment method (MoM) program was written based on [13]. A convergence study indicated that, for the frequency range of interest, 11 TM_{0n} modes and 11 triangular expansion function were sufficient to approximate the aperture field. The program was validated by comparison with [14] (Table 1) and by measurements performed with an HP8510 network analyzer (Figure 2).

The analysis was then performed for twenty normalized frequencies in the range $0.01\le k_0a\le 0.19$ and 56 dielectric constants in the range $1\le\varepsilon'\le 80$, yielding a total of 1120 data points. The MINPACK routine LMDER1 [15], which implements a modified Levenberg-Marquardt algorithm, was used with the merit function:

$$\chi^2=\sum_{i=1}^{1120}\left|\frac{Y_i^{MoM}-Y_i^{Model}}{0.01\times Y_i^{MoM}}\right|^2 \qquad (15)$$

to determine the model parameters. In (14) Y_i^{MoM} and Y_i^{Model} represent the admittance, for $(\omega_i a,\varepsilon'_i)$, computed via MoM and (14), respectively. An acceptable fit ($\chi^2<0.5$) was obtained for N=M=4 and P=Q=8. Figure 3 shows the magnitude of the resulting relative error:

$$\Delta Y=\frac{Y^{Model}-Y^{MoM}}{|Y^{MoM}|}\times 1000 \qquad ppt \qquad (16)$$

for the data used in the fit. Figure 4 shows the magnitude of the corresponding relative error vs. the permittivity for $k_0a=0.19$.

The solution of the inverse problem may be obtained as follows. Define the following functions:

$$b_p=\sum_{n=1}^{4}\hat{\alpha}_{np}(sa)^n \qquad p=1,2,\ldots,8$$

$$b_0=0$$

$$c_q=\sum_{m=1}^{4}\beta_{mq}(sa)^m \qquad q=1,2,\ldots,8$$

$$c_0=1+\sum_{m=1}^{4}\beta_{m0}(sa)^m$$

In terms of these functions, (14) may be rewritten as:

$$\sum_{i=0}^{8}(b_i-Yc_i)\zeta^i=0 \qquad (17)$$

Considering Y to be the measured quantity, the solution of the inverse problem is related via (7) to one of the eight roots of (17). These roots are found by Laguerre's method and selection of the appropriate root is straight-forward. The magnitude of the relative error in the solution of the inverse problem is shown in Figure 5 for the case $k_0a=0.19$. This data was obtained by setting $Y=Y^{MoM}$ in (17) and computing:

$$|\Delta\varepsilon| = \left|\frac{\varepsilon_r^{Model} - \varepsilon_r}{|\varepsilon_r|}\right| \times 100 \quad \% \qquad (18)$$

IV. Conclusions

A new model for the aperture admittance of open-ended waveguide structures was presented. The model is based on the physical and mathematical properties of a passive, one-port driving point admittance. The model is general, applicable to any waveguide/aperture geometry providing the frequency is low enough to ensure single-mode propagation. The model is broadband, no static or quasi-static assumptions are made. The model is explicit in frequency and material permittivity, allowing simple inversion and sensitivity analysis. Finally, the model parameters are easily determined from a relatively small number of computed admittances.

To illustrate the validity of the new model, the case of an open-ended coaxial line was investigated. The admittance data was obtained by a full-wave moment method analysis and includes the effects of radiation and energy storage in the near-field and evanescent waveguide modes. The accuracy of the resulting model for both the direct and inverse problem was demonstrated. The new model should aid in the design and optimization of the transducers for microwave induced hyperthermia, microwave thermography and dielectric spectroscopy.

Table 1
Reflection Coefficient of a 14 mm Coaxial Line at 1 GHz
$a = 2.333mm$, $b = 7.549mm$, $\varepsilon_c = 2.15$, $\varepsilon = 100.0 - j100.0$

Magnitude of reflection coefficient			Phase of reflection coefficient		
$\|\Gamma_\infty\|$ This Work	$\|\Gamma_\infty\|$ Jenkins [14]	$\Delta\|\Gamma_\infty\|$	ϕ^o This Work	ϕ^o Jenkins [14]	$\Delta\phi^o$
0.6709	0.6715	0.0006	-165.52	-165.55	-0.03

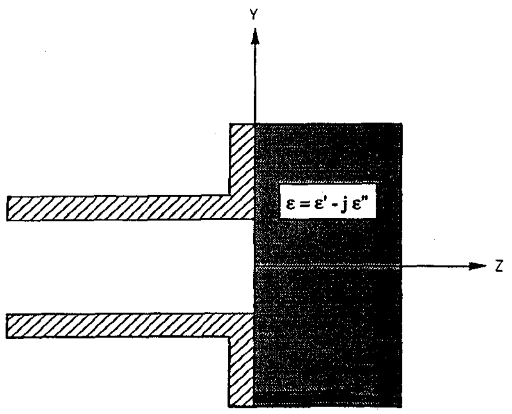

Fig. 1: Geometry of the General Problem

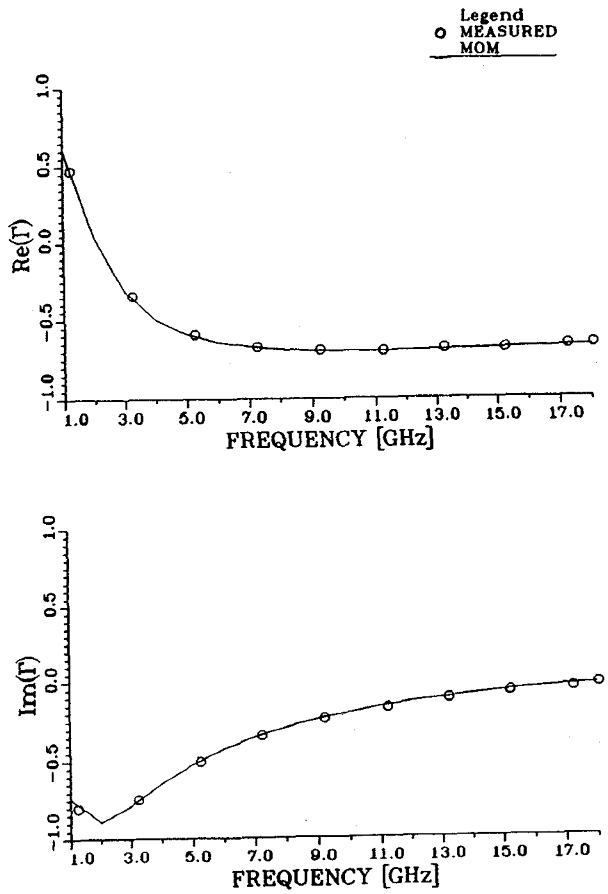

Fig. 2: Aperture Reflection Coefficient of a 3.6mm Coaxial Line Radiating into Water at 25° C

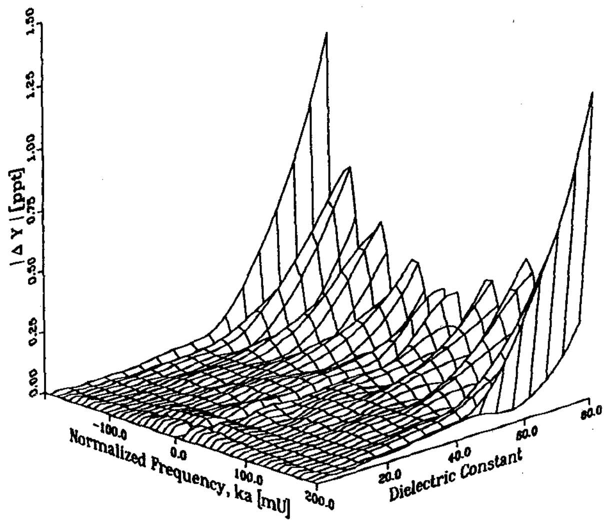

Fig. 3: Magnitude of the Relative Error in Aperture Admittance Computed According to (16)

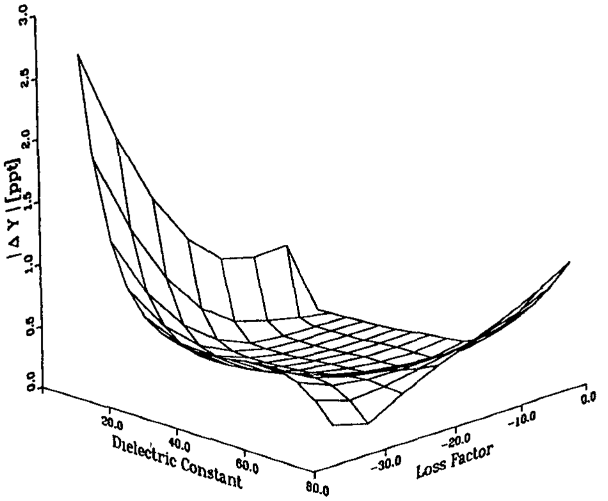

Fig. 4: Magnitude of the Relative Error in the Aperture Admittance for Lossy Dielectrics and a Coaxial Line with $k_0a = 0.19$ Computed According to (16)

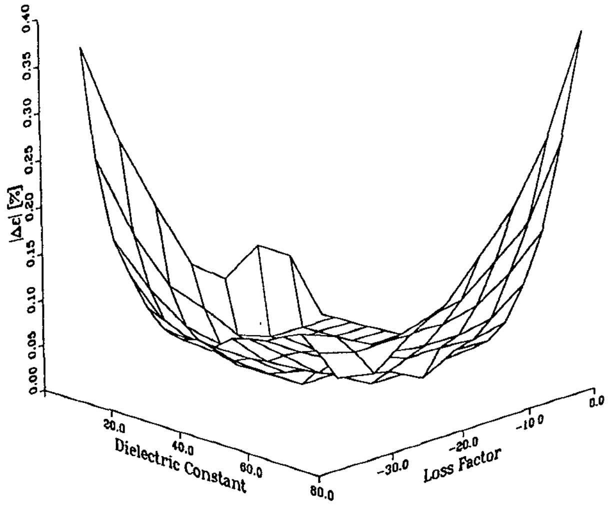

Fig. 5: Magnitude of the Relative Error in the Solution of the Inverse Problem for Coaxial Line with $k_0a = 0.19$ Computed According to (18)

References

1. Von Hipple A.R., *Dielectric Materials and Applications*, New York, MIT Technology Press and Wiley, 1954.
2. Altschuler H.M., "Dielectric Constant," in *Handbook of Microwave Measurements*, Vol. II, M. Sucher and J. Fox, Eds., Brooklyn, N.Y.:Polytechnic Press, 1963, pp. 495-548.
3. Busey H.E., "Measurement of RF Properties of Materials - A Survey," Proc. IEEE, Vol. 55, No. 6, pp. 1046-1053, June 1967.
4. Tanabe E., Joines W.T., "A Nondestructive Method for Measuring the Complex Permittivity of Dielectric Materials at Microwave Frequencies Using an Open Transmission Line Resonator," IEEE Trans. Instrum. Meas., Vol. IM-25, No. 3, pp. 222-226, September 1976.
5. Burdette E.C., Cain F.L., Seals J., "In Vivo Probe Measurement Technique for Determining Dielectric Properties at VHF through Microwave Frequencies," IEEE Trans. Microwave Theory Tech., Vol. MTT-28, No. 4, pp. 414-427, April 1980.
6. Stuchly M.A., Stuchly S.S., "Coaxial Line Reflection Method for Measuring Dielectric Properties of Biological Substances at Radio and Microwave Frequencies - A Review," IEEE Trans. Instrum. Meas., Vol. IM-29, pp. 176-183, 1980.
7. Brady M.M., Symons S.A., Stuchly S.S., "Dielectric Behavior of Selected Animal Tissues In Vitro at Frequencies from 2 to 4 GHz," IEEE Trans. Biomed. Eng., Vol. BME-28, No. 3, pp. 305-307, March 1981.
8. Gajda G., Stuchly S., "An Equivalent Circuit of an Open-Ended Coaxial Line," IEEE Trans. Instrum. Meas., Vol. IM-32, No. 4, pp. 506-508, December 1983.
9. Misra D.K., "A Quasi-Static Analysis of Open-Ended Coaxial Lines," IEEE Trans. Microwave Theory Tech., Vol. MTT-35, No. 10, pp.925-928, October 1987.
10. Deming Xu, Liping Liu, Zhiyan Jiang, "Measurement of the Dielectric Properties of Biological Substances Using an Improved Open-Ended Coaxial Line Resonator," IEEE Trans. Microwave Theory Tech., Vol. MTT-35, No. 12, pp. 1424-1428, December 1987.
11. Guillemin E.A., *Synthesis of Passive Networks, Theory and Methods Appropriate to the Realization and Approximation Problems,* John Wiley & Sons, New York, 1957.
12. Sinclair G., "Theory of Models of Electromagnetic Systems," Proc. IRE, Vol. 36, pp. 1364-1370, November 1948.
13. Mautz J.R., Harrington R.F., "Transmission from a Rectangular Aperture into Half Space through a Rectangular Aperture," Rep. TR-76-5, Dept. of Electrical and Computer Engineering, Syracuse University, New York, 1976.
14. Jenkins S., Preece A.W., Hodgetts T.E., Symm G.T., Warham A.G.P., Clark R.N., "Comparison of Three Numerical Treatments for the Open-Ended Coaxial Line Sensor," Electron. Lett., Vol. 26, No. 4, pp. 234-236, February 1990.
15. Garbow B.S., Hillstrom K.E., More J.J., " MINPACK, Double Precision Version," Argonne National Laboratory, March 1980.